KSME Journal, Vol. 7, No. 3, pp. 231~241, 1993

A Quasistatic Manipulation for Multifingered Robotic Hands

Nak Young Chong*, Donghoon Choi* and Il Hong Suh*

(Received December 10, 1992)

A generalized algorithm for the motion/force planning of the multifingered hand is proposed to generate finite displacements and changes in orientation of objects by considering sliding contacts as well as rolling contacts between the fingertips and the object at the contact point. Specifically, an optimization problem is firstly formulated and solved to find joint velocities and contact forces to impart a desired motion to the object at each time step. Then the relative velocity at the contact point is found by calculating velocity of the fingertip and the velocity of the object at the contact point. Finally, time derivatives of the surface variables and the contact angle of the fingertip and the object at the present time step are computed using the Montana's contact equation to find the contact parameters of the fingertip and the object at the next time step. To show the validity of the proposed algorithm, a numerical example is illustrated by employing the robotic hand manipulating a circular cylinder with three fingers each of which has four joints.

Key Words: Motion/Force Planning, Multifingered Hand, Rolling Contact, Sliding Contact, Nonlinear Programming, Relative Velocity, Surface Variable, Contact Angle, Contact Equation

Nomenclature

- $A_{\beta,\alpha}$: The rotation matrix of a coordinate frame $\{C_\beta\}$ with respect to a coordinate frame$\{C_\alpha\}$
- C_b : Body coordinate frame
- C_{bi} : The local frame of object at the i-th contact point
- C_{fi} : The finger frame fixed to the last link of the finger
- C_{li} : The local frame of the finger at the i-th contact point
- C_p : Reference frame
- F : Contact force
- G : Grasp matrix
- J : Jacobian
- K : Curvature form
- M : Metric
- m : The number of joints
- p : The position vector of contact point
- q : Jont variable
- $\dot{q}$: Joint velocity
- $\ddot{q}$: Joint acceleration
- R_{sr} : Slide/Roll ratio
- R_ψ : The orientation matrix of the x-and y-axes of C_{li} with respect to the x-and y-axes of C_{fi}
- $r_{\beta,\alpha}$: The position vector of a coordinate frame $\{C_\beta\}$ with respect to a coordinate frame $\{C_\alpha\}$
- $\tilde{T}$: Resultant force and moment
- T : Torsion form
- u : Surface variable
- v_x, v_y, v_z : Translational relative velocity
- δ : Slide/Roll mode parameter
- μ : Friction coefficient
- ψ : Contact angle
- $\omega_x, \omega_y, \omega_z$: Rotational relative velocity

* Department of Mechanical Engineering, Hanyang Unirersity, Seoul 133-791, Korea.

1. Introduction

In recent years, dexterous multifingered robotic hands have become of interest as fine manipulations are required for more sophisticated tasks in robot applications. Various multifingered robotic hands have been designed and manufactured and many research works including basic analysis of kinematics and control for stable grasping have also been performed. Another important problem arising from the study of multifingered hands is how to impart finite displacements and/or changes of orientation to a grasped object.

Several research works on such issues have been proposed, where most of them consider only rolling contacts between the fingertips and the object due to the difficulties in finding the evolution of contact points, even though the object could be manipulated more efficiently by allowing sliding contacts at the contact point. Kerr(1985) discussed how to move each finger in order to execute a finite displacement of the object. Kinematic equations are derived from the rolling constraint that the fingertip and object velocities are equal at the contact point. Montana(1988) and Cai and Roth(1987) independently studied the kinematic relations of rigid bodies that maintain contact while in relative motion. The kinematic equations for the contact point evolution were derived. They did not, however, consider the effects of the kinematics of a finger attached to the fingertip. Cole et al.(1988) derived the kinematics of rolling contact for two arbitrary shaped surfaces rolling on each other and presented a scheme for the control of these hands. Cole et al.(1989) also considered the problem of dynamic control of a multifingered hand and presented a new control law that applies specifically to the situation of a hand manipulating a grasped object while certain prespecified fingers slide along the object surface. Brock(1988) derived a kinematic relation between the object motion, the constraints of motion, and the grasp forces. Based on this relation, a method of reorienting a grasped object is proposed. Fearing (1986) considered slip from a quasi-static viewpoint to achieve grasp stability. To the authors' knowledge, no previous work has been reported to positively utilize sliding contacts in the manipulation of the object by multifingered hands.

In this paper, we propose a generalized motion/force planning algorithm for multifingered hands manipulating an object of arbitrary shape considering general relative motions between the fingertip and the object at the contact point. The joint velocities and contact forces are obtained by solving a nonlinear optimization problem given initial contact parameters which is defined as the position vectors and the rotation matrices of the coordinate frame attached to the contact point with respect to the body coordinate frame. The relative velocities then can be determined by calculating the object and fingertip velocities at the contact point. The contact point evolution at the next time step is also determined by utilizing the Montana's contact equation (Montana, 1988) and obtained relative velocities to update the contact parameters. A simulation is finally illustrated by employing a three fingered robotic hand manipulating a circular cylinder to evaluate the validity of the proposed algorithm.

In the following section, motion/force planning problems for multifingered robotic hands are formulated. In section 3, kinematics of multifingered hands grasping an object is described and the kinematics of contact is also described in Section 4. In Section 5, the motion/force planning is shown to be equivalent to finding joint velocities and contact forces for each finger. Simulation results are summarized in Section 6 and conclusions are drawn in the final section.

2. Problem Statement

A motion/force planning for multifingered hands manipulating an object can be achieved by hierarchically solving the PROBLEM I and II at each time step.

(PROBLEM I) Find the joint velocities and contact forces of the fingers minimizing (or maximizing) a performance index to generate desired motions of the object satisfying the dynamic force/moment equilibrium equation, the compatibility equation of relative motions, and Coulomb's law of friction as well as some physical constraints given contact parameters at the current time step.

The contact parameters are changed according to the changes in the contact geometries of the fingertips and the object resulted from the changes in the configurations of the finger joints and the object. Let the surface variables be defined as the parameters which indicate the contact point on the surface (Fig. 1). Also let the contact angle be defined as the angle between the corresponding axes of two coordinate frames in the common tangent plane attached to respective contact points of two contacting bodies. Then, the contact parameters evolve by updating the surface variables and the contact angle in response to a relative motion of the fingertip and the object.

(PROBLEM II) Find the time derivatives of the surface variables and the contact angle of the object and the fingertips at present time step to predict the contact parameters at the next time step.

Thus, PROBLEM I is iteratively solved at each time step under the contact parameters to be found from PROBLEM II. To the authors' knowledge, this strategy is definitely the first one to date for planning the motions and contact forces of multifingered hands manipulating an object with sliding and rolling contacts are simultaneously considered.

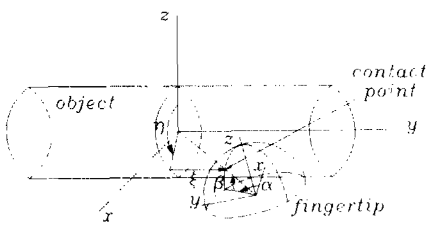

Fig. 1 The surface variables of the fingertip and the object

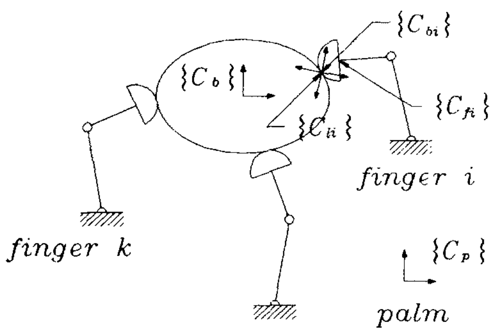

Fig. 2 A k-fingered robotic hand grasping an object

3. Kinematics of Multifingered Hand

In this section, kinematics of multifingered robotic hands mainly adapted from Li's work (Li, 1989) is described. A k-fingered hand grasping an object is shown in Fig. 2. Let the number of joints and the joint variables of finger i, $i=1, \cdots, k$, be denoted as m_i and $q_i \in R^{m_i}$, respectively. To describe the relative motions between a fingertip and an object, a set of coordinate frames are defined as follows: The reference frame, $\{C_p\}$, is fixed to the palm of the hand; the body coordinate frame, $\{C_b\}$, is fixed to the mass center of the object; the finger frame, $\{C_{fi}\}$, is fixed to the last link of finger i; at the i-th point of contact between the finger i and the object, the local frame of the object, $\{C_{bi}\}$, is fixed with respect to $\{C_b\}$ and the local frame of the finger i, $\{C_{li}\}$, is fixed with respect to $\{C_{fi}\}$, where their z-axes coincide with the outward pointing normal to the object surface and the fingertip surface, respectively and their x-and y-axes lie in the common tangent plane as well as they share a common origin at the contact point.

Let $r_{\beta,\alpha} \in R^3$ and $A_{\beta,\alpha} \in SO(3)$ denote the position vector and the rotation matrix of a coordinate frame $\{C_\beta\}$ with respect to a coordinate frame $\{C_\alpha\}$, respectively. If $(r_{\beta,\alpha(t)}, A_{\beta,\alpha(t)})$ is any curve in $SE(3) \equiv R^3 \times SO(3)$ representing the trajectory of $\{C_\beta\}$ with respect to $\{C_\alpha\}$, the translational and rotational velocities of $\{C_\beta\}$ with respect to $\{C_\alpha\}$ can be described by

$$v_{\beta,\alpha}=A^t_{\beta,\alpha}\dot{r}_{\beta,\alpha} \text{ and } \omega_{\beta,\alpha}=S^{-1}(A^t_{\beta,\alpha}\dot{A}_{\beta,\alpha}), \tag{1}$$

where S is an operator defined by

$$\omega=\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\end{pmatrix},\ S(\omega)=\begin{pmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{pmatrix}, \tag{2}$$

and superscript t implies the transpose.

For any three coordinate frames $\{C_\alpha\}$, $\{C_\beta\}$, and $\{C_\gamma\}$, the following relation holds between their relative velocities:

$$v_{\gamma,\alpha}=A^t_{\gamma,\beta}(v_{\beta,\alpha}+\omega_{\beta,\alpha}\times r_{\gamma,\beta})+v_{\gamma,\beta} \tag{3.1}$$

$$\omega_{\gamma,\alpha}=A^t_{\gamma,\beta}\omega_{\beta,\alpha}+\omega_{\gamma,\beta} \tag{3.2}$$

In particular, when $\{C_\gamma\}$ is fixed with respect to $\{C_\beta\}$, the velocity $\{C_\gamma\}$ is related to that of $\{C_\beta\}$ by a constant tranformation, given by,

$$\begin{pmatrix} v_{\gamma,\alpha} \\ \omega_{\gamma,\alpha} \end{pmatrix}=\begin{pmatrix} A^t_{\gamma,\beta} & -A^t_{\gamma,\beta}S(r_{\gamma,\beta}) \\ 0 & A^t_{\gamma,\beta} \end{pmatrix}\begin{pmatrix} v_{\beta,\alpha} \\ \omega_{\beta,\alpha} \end{pmatrix} \tag{4}$$

Let (v^i_x, v^i_y, v^i_z) and $(\omega^i_x, \omega^i_y, \omega^i_z)$ denote the translational and rotational velocities of $\{C_{bi}\}$ with respect to $\{C_{li}\}$, respectively. These are velocities of the object with respect to finger i expressed in local frames. Also let the contact angle defined as the angle between the x-axes of $\{C_{bi}\}$ and $\{C_{li}\}$ be denoted as $\{\psi_i\}$. We choose the sign of $\{\psi_i\}$ so that a rotation of $\{C_{bi}\}$ through $-\psi_i$ around its z-axis aligns the x-axes. Using (3), then, the velocity of $\{C_{bi}\}$ can be expressed as

$$\begin{pmatrix} v_{bi,p} \\ \omega_{bi,p} \end{pmatrix}=\begin{pmatrix} A_{\psi_i} & 0 \\ 0 & A_{\psi_i} \end{pmatrix}\begin{pmatrix} v_{li,p} \\ \omega_{li,p} \end{pmatrix}+\begin{pmatrix} v^i_x \\ v^i_y \\ v^i_z \\ \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{pmatrix}, \tag{5}$$

where

$$A_{\psi_i}=\begin{pmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ -\sin\psi_i & -\cos\psi_i & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{6}$$

is the orientation matrix of $\{C_{bi}\}$ with respect to $\{C_{li}\}$.

On the other hand, the velocity of $\{C_{bi}\}$ is related to the velocity of $\{C_b\}$ by

$$\begin{pmatrix} v_{bi,p} \\ \omega_{bi,p} \end{pmatrix}=\begin{pmatrix} A^t_{bi,b} & -A^t_{bi,b}S(r_{bi,b}) \\ 0 & A^t_{bi,b} \end{pmatrix}\begin{pmatrix} v_{b,p} \\ \omega_{b,p} \end{pmatrix}, \tag{7}$$

and similarly one has for finger i that

$$\begin{pmatrix} v_{li,p} \\ \omega_{li,p} \end{pmatrix}=\begin{pmatrix} A^t_{li,fi} & -A^t_{li,fi}S(r_{li,fi}) \\ 0 & A^t_{li,fi} \end{pmatrix}\begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix} \tag{8}$$

Moreover, the velocity of the finger frame, $\{C_{fi}\}$, is related to the velocity of the finger joints, $\dot{q}_i$, by the finger Jacobian,

$$\begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix}=J_i(q_i)\dot{q}_i \tag{9}$$

Let J_{iu} and J_{il} imply the upper and lower matrices of the finger Jacobian, respectively. Finally, then, the translational and rotational relative velocities of the object with respect to fingertip at the i-th contact point can be expressed in terms of the finger Jacobian and the velocity of the finger joints as well as the contact parameters given the desired object motions $v_{b,p}$ and $\omega_{b,p}$ by substituting Eqs. (7),(8), and (9) into Eq. (5).

$$\begin{pmatrix} v^i_x \\ v^i_y \\ v^i_z \end{pmatrix}=[A^t_{bi,b}v_{b,p}-A^t_{bi,b}S(r_{bi,b})\omega_{b,p}]$$
$$-[A_{\psi_i}A^t_{li,fi}J_{iu}\dot{q}_i-A_{\psi_i}A^t_{li,fi}S(r_{li,fi})J_{il}\dot{q}_i], \tag{10.1}$$

$$\begin{pmatrix} \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{pmatrix}=[A^t_{bi,b}w_{b,p}]-[A_{\psi_i}A^t_{li,fi}J_{il}\dot{q}_i], \tag{10.2}$$

where v^i_x and v^i_y represent sliding, ω^i_x and ω^i_y rolling, and ω^i_z spin motions, respectively.

4. Montana's Kinematic Equations of Contact

This section describe the motion of a point of contact over the surfaces of two contacting object in response to a relative motion of these objects. When the fingertips roll or slide over the object, the contact parameters $(r_{bi,b}, A_{bi,b})$ of the object and $(r_{li,fi}, A_{li,fi})$ of the finger evolve according to the Montana's kinematic equations of contact(Montana, 1988). If the fingertip and the object surfaces are parameterized by the surface variables u (α and β for the fingertip and η and ξ for the object), we can describe the contact parameters of fingertip and object by these variables (Fig. 1). Let the symbols K, T, and M represent, respectively, the curvature form, torsion form, and metric at time t at the point of contact with respect to its coordinate system (O'Neill, 1966). Let R_ψ represent the orientation matrix of the x-and y-axes of $\{C_{li}\}$ with respect to the x-and y-axes of $\{C_{fi}\}$ and the subscripts o and f denote the object and fingertip, respectively. Also let $\widetilde{K}_f$ be defined as $R_\psi K_f R_\psi$ and let $K_o+\widetilde{K}_f$ be the relative curvature form. At a point of contact, if the relative curvature form is invertible, then the point of contact and the angle of contact evolve according to

$$\dot{u}_o = M_o^{-1}(K_o + \tilde{K}_f)^{-1}([-\omega_y \omega_x]^t - \tilde{K}_f [v_x\ v_y]^t), \tag{11}$$

$$\dot{u}_f = M_f^{-1} R_\psi (K_o + \tilde{K}_f)^{-1}([-\omega_y \omega_x]^t + K_o [v_x\ v_y]^t), \tag{12}$$

$$\dot{\psi} = \omega_z + T_o M_o \dot{u}_o + T_f M_f \dot{u}_f, \tag{13}$$

$$0 = v_z \tag{14}$$

Thus, the contact equations give time derivatives of the surface variables and the contact angle by receiving the relative velocities of two contacting object(Fig. 3). Equations (11) thru(13) are called the first, second, and third contact equations, respectively. Equation (14) is the kinematic constraint of contact imposing the constraint on the relative motion necessary to maintain contact.

Montana imposed the constraints on the above equations that $v_x = v_y = \omega_z = 0$ such that the fingers must roll without slipping (Montana, 1988), which is thought to be unreasonable in the sense of manipulating an object with multifingered hands. It may be impractical to manipulate an object using only rolling contacts and the grasping stability may be maintained even though one of the fingers are exposed to sliding contacts. Thus, we will handle the contact equations not dropping any terms to consider the general relative velocities which can be obtained by using

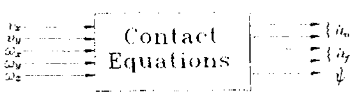

Fig. 3 The relative velocities and contact equations

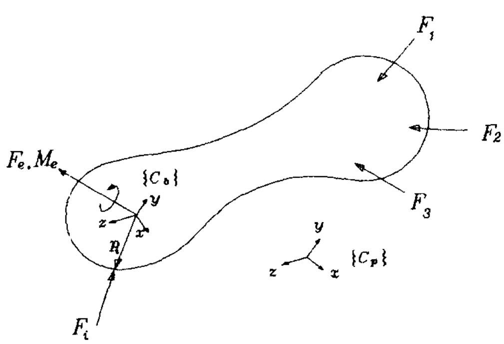

Fig. 4 Modelling of a manipulation of an object by multifingered robotic hands

Eqs. (10.1) and (10.2).

5. Solution Approaches for RROBLEM I: Nonlinear Programming Approach

5.1 Force/Moment equilibrium equation

In Fig. 4, all vectors are represented with respect to the object coordinate system $\{C_b\}$ and a frictional point contact model is assumed at the contact point. Let F_i be the force vectors applied to the object at i-th contact point by each finger and $\tilde{T} = [F_e^t\ M_e^t] \in R^{6\cdot 1}$, denote the resultant force and moment vectors, respectively. Let p_i and n be the position vector from the origin of the object coordinate system to i-th contact point and the number of contact points, respectively. Then, the force/moment equilibrium equation can be written as follows.

$$F_e = \sum_{i=1}^{n} F_i, \tag{15.1}$$

and

$$M_e = \sum_{i=1}^{n} p_i \times F_i \tag{15.2}$$

Equation (15) can be written in the matrix form as

$$\tilde{T} = GF, \tag{16}$$

where $G \in R^{6\times 3n}$ is defined by

$$G \equiv \begin{pmatrix} I_3 & I_3 & \cdots & I_3 \\ P_1 & P_2 & \cdots & P_n \end{pmatrix}, \tag{17}$$

and is time dependent as the contact parameters evolve. Here I_3's are 3×3 unit matrices and P_i are the 3×3 skew symmetric matrices with zero diagonal elements equivalent to the vector product of position vectors $p_i = (p_{ix}, p_{iy}, p_{iz})^t$ shown as

$$P_i \equiv \begin{pmatrix} 0 & -p_{iz} & p_{iy} \\ p_{iz} & 0 & -p_{ix} \\ -p_{iy} & p_{ix} & 0 \end{pmatrix} \tag{18}$$

It is noted that the dynamic equilibrium is also maintained by using above static force/moment equilibrium equation if the inertia force equal to the product of the mass of the object and its acceleration and directed oppositely to the acceleration is added to $\tilde{T}$.

5.2 Forces transmitted at a point of contact

The resultant force transmitted from one surface to another through a point of contact is resolved into a normal force F_n acting along the common normal, which generally must be compressive, and a tangential force F_t in the tangent plane sustained by friction. The magnitude of F_t must be less than or, in the limit, equal to the force of limiting friction, i. e.

$$F_t \leq \mu F_n, \tag{19}$$

where μ is the coefficient of limiting friction.

5.3 Contact maintenance condition

If the contact between the surfaces of the fingertip and those of the object is continuous, their velocity components along the common normal must be equal such that the surfaces are neither separating nor overlapping. Thus, v_z^i always equals zero and can be represented in terms of the finger Jacobians and joint velocity vectors as well as contact parameters as follows :

$$[A^t_{bi,b} v_{b,p} - A^t_{bi,b} S(r_{bi,b})\omega_{b,p}]_z - [A_{\psi_i} A^t_{li,fi} J_{iu}\dot{q}_i - A_{\psi_i} A^t_{li,fi} S(r_{li,fi}) J_{il}\dot{q}_i]_z = 0, \tag{20}$$

where subscript z implies z-component. Any motion of contacting surfaces must satisfy the contact maintenance condition and can be regarded as the combination of sliding, rolling and spin.

5.4 Consistency of roll/slide mode between force and motion

When the object is manipulated by the multifingered hand, either rolling or sliding at the contact points may be resulted from the contact forces. To generate the corresponding relative motions for the contact forces applied at the contact point, following mode parameter is defined.

$$\delta = \mu F_n - F_t \tag{21}$$

While the contact forces result in rolling motions if δ is greater than zero, the contact forces generate sliding motions if δ is equal to zero. Let v_t denote the translational relative velocity vector. Then, the consistency of the contact forces and relative motions at the contact points are accomplished by satisfying the following compatibility equation.

$$\delta \cdot v_t = 0 \tag{22}$$

Thus, while v_t should be zero to imply rolling motions if δ is greater than zero, v_t have any nonzero finite magnititudes to get sliding motions if δ is zero. When both quantities are zero at the same time, sliding motions may impend.

5.5 The direction of tangential forces and sliding velocities

The tangential force of friction is constrained to be no greater than the product of the normal force with the coefficient of static friction. In a purely sliding contact the tangential force reaches its limiting value in a direction opposed to the sliding velocity. In this paper, the sliding velocity is defined the translational relative velocities of the object with respect to the fingertip at the contact point. Thus, the direction of the tangential force and sliding velocity should be coincident. Let$\| * \|$ denote the Euclidean norm. Then, the directional condition of tangential force and sliding velocities is described as follows :

$$\frac{F_t}{\|F_t\|} = \frac{v_t}{\|v_t\|} \tag{23}$$

5.6 Nonlinear optimization problem formulation

Now a motion/force planning problem can be formulated into an optimization problem to find the joint velocities and contact forces at each time step given contact parameters satisfying above constraints as well as some physical constraints.

Minimize

$$\Phi, \tag{24}$$

Subject to

$$GF = \tilde{T}, \tag{25}$$

$$F_t \leq \mu F_n, \tag{26}$$

$$v_z^i = 0, \tag{27}$$

$$q_{i\min} \leq q_i \leq q_{i\max}, \tag{28}$$

$$\dot{q}_{i\min} \leq \dot{q}_i \leq \dot{q}_{i\max}, \tag{29}$$

$$\ddot{q}_{i\min} \leq \ddot{q}_i \leq \ddot{q}_{i\max}, \tag{30}$$

$$F_{i\min} \leq F_i \leq F_{i\max}, \tag{31}$$

$$\delta \cdot v_t = 0, \tag{32}$$

$$\frac{F_t}{\|F_t\|} = \frac{v_t}{\|v_t\|} \tag{33}$$

Equation (24) is the user-defined performance index and the candidates for Eq. (24) may be the magnitudes of joint velocities, joint accelerations, and contact forces and so on. Equation (25) is the dynamic force/moment equalibrium equation. It is remarked that the risk of losing contact stability due to the sliding contact does not happen since the object should satisfy the dynamic force/moment equilibrium equation to ensure the force closure condition. Equations (26) and (33) are the Coulomb friction constraints and Eq. (27) is contact maintenance condition. Equations (28), (29) and (30) are joint angle, velocity and acceleration constraints, respectively. Equation (31) is the constraint of the magnitude of the contact force. Equation (32) is the mode compatibility condition of roll or slide between contact force and relative velocities. Thus, given the contact parameters, the contact forces and joint velocities of the fingers are obtained by solving the above nonlinear optimization problem.

The procedure to find the joint velocities and contact forces at each time step can be summarized as follows:

[Step 0] Read the initial configurations, contact parameters at the current time step.

[Step 1] Calculate the object velocity at the contact point by Eq. (7).

[Step 2] Determine the joint velocities and contact forces of the fingers to find the fingertip velocities by solving the nonlinear optimization problem given Eqs. (24) thru (33).

[Step 3] Calculate the relative velocities at the contact point by Eqs. (10.1) and (10.2).

[Step 4] Determine the time derivatives of the surface variables and contact angle at the current time step by the contact equations Eqs. (11) thru (13).

[Step 5] Update the contact parameters and the joint configurations of the fingers.

[Step 6] Go to Step 1.

6. Simulation Results

The re-orienting task of a circular cylinder (Fig. 5) is considered to show the validities of our proposed method for a robotic hand with three fingers each of which has four joints, where the z-axes define the joints' axes of rotation. The specification of the hand is given in Table 1. In Table 2, the specification of the object is summarized. In Table 3, the initial joint configurations of the fingers are given. Initial contact parameters of the fingertip surface and object surface are given

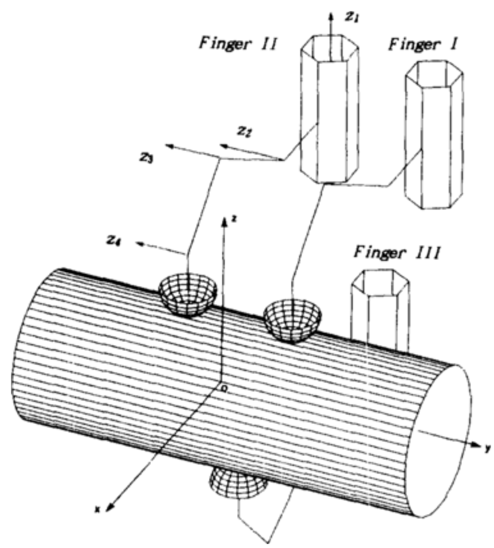

Fig. 5 The re-orienting task of a circular cylinder by a multifingered hand

Table 1 Specification of robotic hand

No. of Fingers		3	
No. of Joints/Finger		4	
Link length of each finger [m]			
link 1 : 0.028	link 2 : 0.062	link 3 : 0.036	link 4 : 0.014
Geometry of fingertip surface			Hemisphere
Radius of fingertip [m]		0.01	

Table 2 Specification of the object

Ceometry of surface	Circular cylinder
Radius [m]	0.0273
Length [m]	0.15
Mass [kg]	0.1
Friction coefficient	0.5
Contact type	Frictional point contact

Table 3 Initial joint configurations of fingers [rad]

	Joint 1	Joint 2	Joint 3	Joint 4
Finger I	0.	0.5236	4.9742	5.4978
Finger II	0.	0.5236	4.9742	5.4978
Finger III	0.	5.7596	1.3090	0.7854

in Table 4. The orientation of reference and body coordinate frames are initially chosen to be coincident. A rotational motion about x-axis of reference frame with a velocity profile as shown in Fig. 6 is given to the object.

The solution of the problem is obtained by utilizing a general purpose nonlinear optimization solver Integrated Design Optimization Library(IDOL) Ver. 1.5 developed at Applied Mechanics & Optimal Design Laboratory in Hanyang University and has been implemented on an IBM RISC/6000 320 H. In IDOL based on the Augmented Lagrange Multiplier Method (Vanderplaats, 1984), several schemes are devised for computational enhancements of the ALM method in the sense of selecting good initial guesses for design variables and Lagrange multipliers with scalings of constraints, restartings of descent vectors, and dynamic stopping criterions. Specifically, descent vectors are determined by using the Broydon-Fletcher-Goldfarb-Shanno (B-FGS) method (Vanderplaats, 1984). For line search, the incremental search method is firstly used to find bounds on the solution, then the bounds are refined by the golden section method, and finally a cubic polynomial approximation technique is applied to interpolate for the solution using the last four function values.

It is remarked that large contact forces might result in low grasping stability, because even a small position error may cause a large disturbing moment at the mass center of the object and the excessive contact forces are not proper for grasping an fragile object. Thus, we may choose Φ in Eq. (24) as $\Phi=\|F_i\|$. Figures 7 thru 9 show the minimum contact forces of each finger to generate a desired object motion. It is observed that the inertia forces are induced during the acceleation and deceleration periods and the weight of the object are mainly sustained by the z-components of the contact forces for Finger III. The joint velocities of the fingers are obtained as shown in

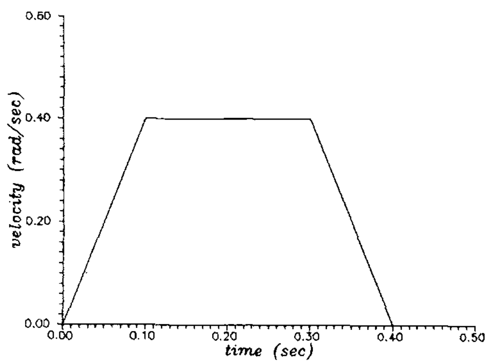

Fig. 6 A velocity profile given to an object

Table 4 Initial surface variable and the contact angle of fingertip and object

	Fingertip		Object		Contact
	α[rad]	β[rad]	η[rad]	ξ[m]	Angle ψ
Contact Point 1	0	1.5708	0	0.02	1.5708
Contact Point 2	0	1.5708	0	−0.02	−1.5708
Contact Point 3	0	1.5708	3.1416	2	3.1416

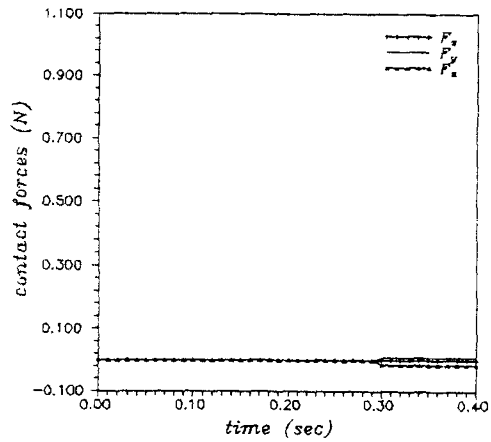

Fig. 7 The minimum contact forces of Figer I

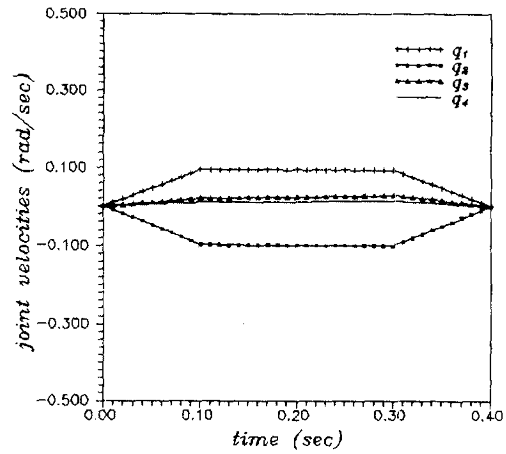

Fig. 10 The joint velocities of Figer I with pure rolling contacts

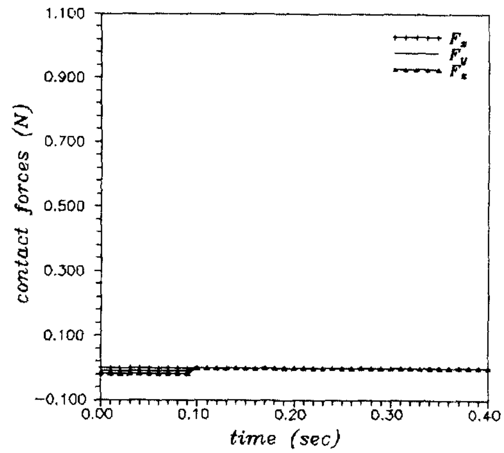

Fig. 8 The minimum contact forces of Figer II

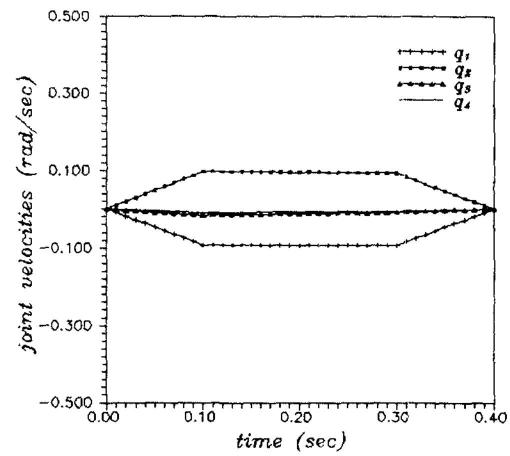

Fig. 11 The joint velocities of Figer II with pure rolling contacts

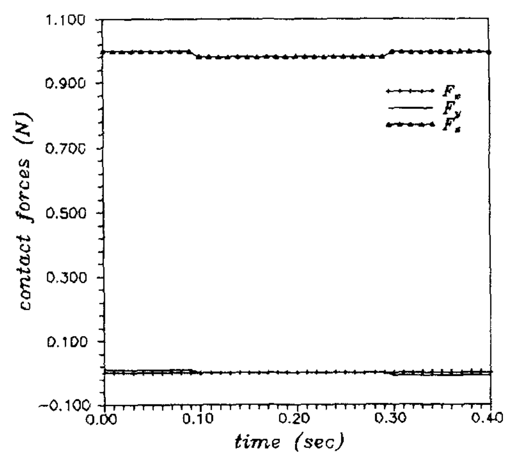

Fig. 9 The minimum contact forces of Figer III

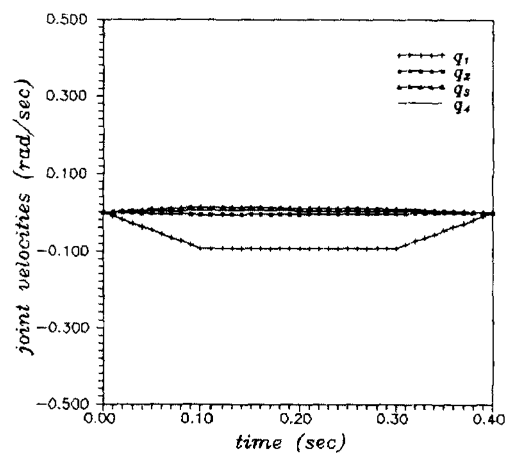

Fig. 12 The joint velocities of Figer III with pure rolling contacts

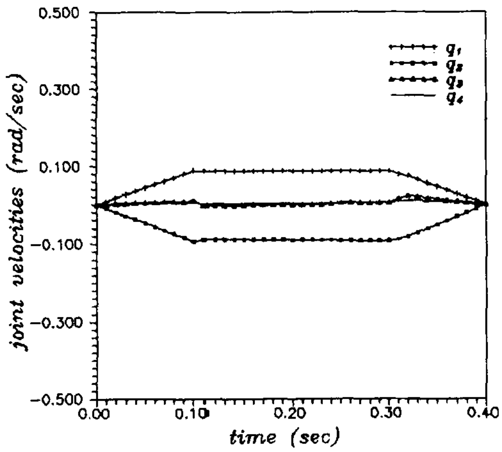

Fig. 13 The joint velocities of Figer I with minimum sliding contacts

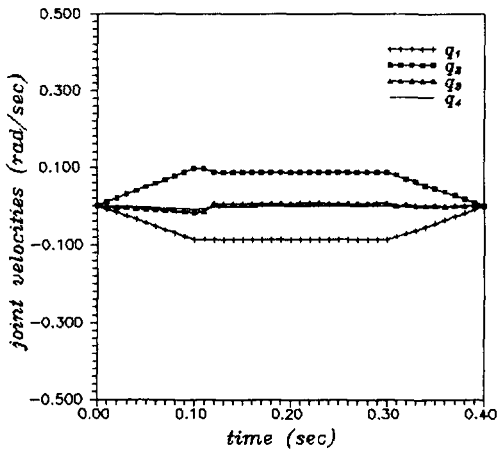

Fig. 14 The joint velocities of Figer II with minimum sliding contacts

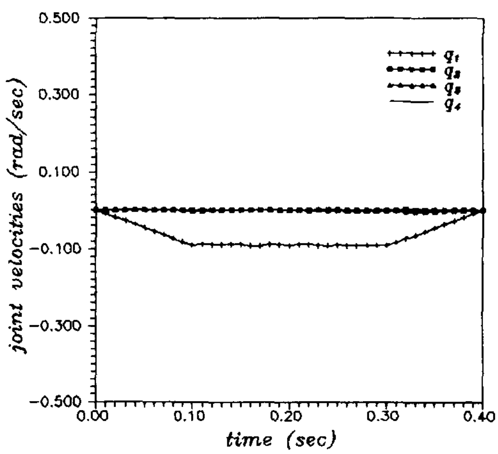

Fig. 15 The joint velocities of Figer III with minimum sliding contacts

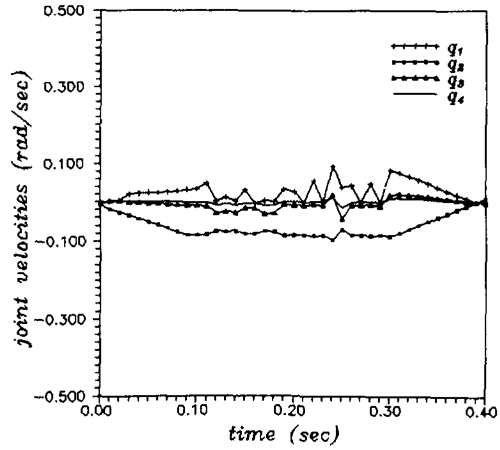

Fig. 16 The minimum joint velocities of Finger I

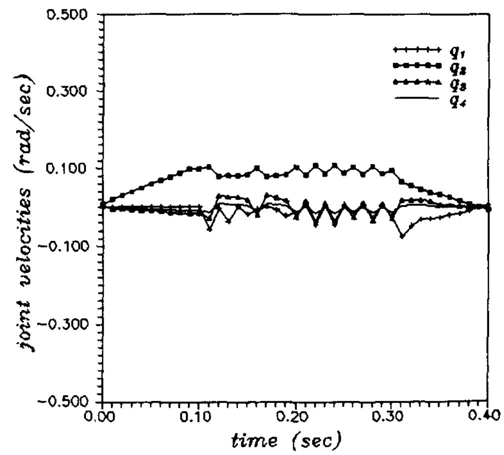

Fig. 17 The minimum joint velocities of Finger II

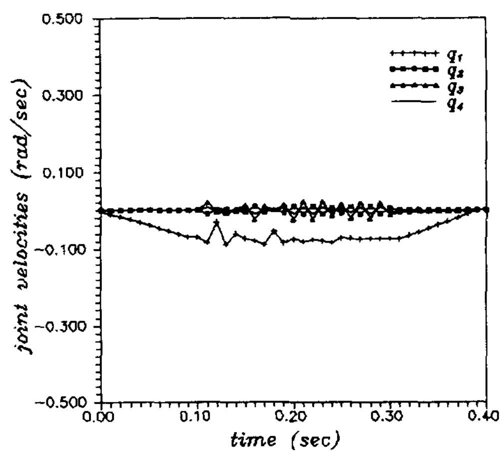

Fig. 18 The minimum joint velocities of Finger III

Figs 10 thru 12, under the assumption that the fingertips always roll without sliding contacts previously reported in many research works (Kerr, 1985, Montana, 1988, Cole, 1988). Similar results can also be obtained as shown in Figs. 13 thru 15 by our proposed algorithm when choosing Φ as $\Phi=\|v_t\|$ which implies sliding velocities at each time step.

If one is interesting to the minimum magnitude of joint velocities with which the maximum manipulability of fingertip and minimum energy consumption together with minimum contact forces are expected to be obtained, one can obtain such minimum joint velocities as shown in Figs. 16 thru 18 by choosing the performance index as $\Phi=\|\dot{q}_i\|+\|F_i\|$. If one would like to get minimum joint accelerations, a different result on joint velocity can be obtained by choosing Φ as $\Phi=\|\ddot{q}_i\|$. The contact points can be forced to evolve more appropriately for a given object motion by using the extra $d.o.f.$'s generated from sliding contacts. This may be useful in re-positioning contact points on the object when good initial contact points are not available. Thus, our proposed generalized contact motions seem to be rather beneficial than other contact motions only using rolling contacts, in the sense that several different desired motions can be generated by properly choosing a performance index, while contact motions only using rolling contacts are uniquely generated for a given motion regardless of a performance index.

7. Conclusion

A generalized motion/force planning algorithm for multifingered robotic hands manipulating an object of arbitrary shape was proposed. In this study, the general relative motions at the contact point are considered to include all kinds of contact motions. The contact forces and joint velocities to generate a desired object motion were found by utiling a nonlinear optimization technique. A simulation was presented by employing a three-fingered robotic hand re-orienting a circular cylinder. The derivation of useful performance indices for motion/force plannings of multifingered hands is the subject of our future research.

References

Brock, D.L., 1988, "Enhancing the Dexterity of a Robot Hand using Controlled Slip," IEEE Int. Conf. on Robotics and Automation, pp. 249 ~251.

Cai, C. and Roth, B., 1987, "On the Spatial Motion of a Rigid Body with Point Contact," IEEE Int. Conf. on Robotics and Automation, pp. 686~694.

Cole, A.A., Hauser, J. and Sastry, S, 1988, "Kinematics and Control of Multifingered Hands with Rolling Contact," IEEE Int. Conf. on Robotics and Automation, pp. 228~233.

Cole, A.A., Hsu, P. and Sastry, S, 1989, "Dynamic Regrasping by Coordinated Control of Sliding for a Multifingered Hands," IEEE Int. Conf. on Robotics and Automation, pp. 228 ~233.

Fearing, R.S., 1986, "Implementing a Force Strategy for Object Re-Orientation," IEEE Int. Conf. on Robotics and Automation, pp. 96~102.

Kerr, J.R., 1985, *An Analysis of Multi-Fingered Hands*, Ph. D. Dissertation, Mechanical Engineering, Stanford University.

Li, Z., Hsu, P. and Sastry, S., 1989, "Grasping and Coordinated Manipulation by a Multifingered Robot Hand," The Int. Journal of Robotics Research, Vol. 8, No. 4, pp. 33~50.

Montana, D.J., 1988, "The Kinematics of Contact and Grasp," The Int. Journal of Robotics Research, Vol. 7, No. 3, pp. 17~32.

O'Neill, B., 1966, *Elementary Differential Geometry*, Academic Press.

Vanderplaats, G.N., 1984, *Numerical Optimization Techniques for Engineering Design with Applications*, McGraw-Hill Book Company.